Primary τ-Invariants in the UNNS Substrate

Closure, Relaxation, and Projection as Irreducible Structural Principles

UNNS Research Collective

Abstract

We formalize the notion of *primary τ-invariants* within the UNNS (Unbounded Nested Number Sequences) substrate. Primary τ-invariants are constants arising from irreducible closure constraints of recursive structure. They are not generated by operators, but are identified by their persistence through collapse. We show that exactly three such invariants exist: $\sqrt{2}$, e, and π, corresponding respectively to orthogonal closure, relaxation toward equilibrium, and projection geometry. All other constants are shown to be derived structures composed from these primary invariants.

1 Introduction

Mathematical and physical constants recur across domains with remarkable consistency. Within the UNNS framework, this recurrence is not attributed to numerical coincidence, but to structural inevitability. Constants are treated not as primitive numbers, but as manifestations of closure constraints in recursive organization.

This paper identifies the minimal set of constants that arise directly from irreducible closure principles of the substrate and establishes their status as *primary τ-invariants*.

2 Structural Regimes of the UNNS Substrate

The UNNS framework distinguishes between *structural regimes* and *operators*.

The symbols Φ, Ψ, and τ do not denote operators. They label successive structural regimes of recursive organization:

- Φ-regime: discrete generability via finite enumeration.
- Ψ-regime: relational consistency via ratios, equivalences, and symmetry.
- τ-regime: closure constraints that cannot be reduced to finite generation.

These regimes describe how structure is organized, not actions performed on it.

3 Operators and Collapse (Codex Alignment)

Separately, the UNNS Operator Codex defines a sequence of numbered operators acting on recursive states. Among these, **Operator XII (Collapse / Sobra–Sobtra)** plays a unique role.

Operator XII is a destructive, terminal operator. Its function is to eliminate unstable or residual recursive structures and to return recursion toward the Zero state. Operator XII does not generate structure, does not define closure, and does not participate in the Φ–Ψ–τ structural progression.

Instead, Operator XII acts *after* closure by absorbing what cannot remain consistent.

4 Definition of τ-Invariant

Definition 1 (τ-Invariant)

A structure is called a τ-invariant if it satisfies a closure constraint that persists under arbitrary Φ-refinement and remains consistent at the τ-regime.

Definition 2 (Primary τ-Invariant)

A constant C is a *primary τ-invariant* if and only if:

1. C arises from a single, irreducible closure constraint in the τ-regime.
2. The closure constraint defining C does not depend on any other invariant.
3. C remains invariant under arbitrary Φ-refinement.
4. C survives the action of Operator XII, which eliminates all non-closed or unstable structures.

Operator XII does not produce τ-invariants. It reveals them by destroying everything else.

5 The τ-Basis Theorem (Mechanism Completeness)

This section establishes the foundational result of the UNNS Substrate: that the space of irreducible closure mechanisms is complete and admits no extension beyond three mutually exclusive types. This result concerns *closure principles*, not numerical constants, and therefore avoids classification by empirical enumeration.

5.1 Preliminary Definitions

[Closure Constraint] A *closure constraint* is a rule that enforces global consistency on a structure generated under -refinement, such that the constraint cannot be satisfied by any finite -construction but admits arbitrarily precise approximations.

[-Refinement Persistence] A closure constraint is said to *persist under -refinement* if, for any sequence of -level approximations, the residual error of the constraint monotonically decreases without admitting finite termination.

[Irreducible Closure Mechanism] An *irreducible closure mechanism* is a closure constraint that:

1. persists under -refinement,
2. survives projection by Operator XII,
3. cannot be decomposed into a composition of other closure constraints.

These definitions are purely structural and make no reference to specific numerical values.

5.2 Mechanism Completeness Theorem

[Mechanism Completeness] Every irreducible closure mechanism admissible in the UNNS Substrate belongs to exactly one of the following three and only three classes:

1. **Orthogonal Closure**: enforcement of magnitude consistency under independent contributions.
2. **Relaxational Closure**: enforcement of equilibrium under iterative normalization.
3. **Projective Closure**: enforcement of global consistency under cyclic or angular projection.

No fourth irreducible closure mechanism exists.

5.3 Proof Sketch

The proof proceeds by exhaustion over admissible closure behaviors.

Let C be an irreducible closure mechanism.

- If C enforces consistency across independent components, it reduces to orthogonal magnitude closure.
- If C enforces convergence through iterative normalization, it reduces to relaxational equilibrium.
- If C enforces consistency via cyclic or global projection, it reduces to projective closure.

Any closure behavior not belonging to one of these classes can be expressed as a composition of them, violating irreducibility. Therefore, the set is complete.

5.4 Canonical Representatives

Each closure mechanism admits a minimal canonical representative:

- $\sqrt{2}$ for orthogonal closure,
- e for relaxational closure,
- π for projective closure.

These constants are *representatives*, not defining elements. The theorem does not depend on their numerical properties.

5.5 Derived Structures and Composition

[Derived τ-Structure] A *derived τ-structure* is a quantity produced by admissible compositions of irreducible closure mechanisms. Such structures may be stable under Operator XII but do not constitute new closure classes.

Admissible composition operations are limited to:

- orthogonal composition,
- sequential relaxation,
- projection chaining.

Operations such as exponentiation or limits do not generate new closure mechanisms; they only produce derived structures within the existing basis.

5.6 Relation to Operator XII

Operator XII acts as a terminal projection that eliminates unstable or inconsistent structures while preserving irreducible closure constraints. It does not generate closure mechanisms but reveals their persistence by destruction of non-admissible structures.

5.7 Empirical Interpretation

While the theorem is structural, its empirical relevance lies in the observation that physical systems repeatedly instantiate these closure mechanisms across domains (geometry, equilibration, projection-based measurement). The appearance of $\sqrt{2}$, e, and π in physical law reflects the substrate's closure requirements rather than numerical coincidence.

5.8 Non-Circularity Statement

The classification does not proceed by identifying constants and inferring mechanisms. Instead, mechanisms are defined independently, and constants appear only as minimal representatives. This removes circularity and prevents post-hoc classification.

5.9 Corollary (No Basis Extension)

[Derived τ-Structures] Any τ-invariant constant $c \notin \{\sqrt{2}, e, \pi\}$ is a *derived τ-structure* whose apparent stability arises from the composition of one or more primary closure mechanisms.

Such structures may exhibit:

- stable fixed points under finite recursion depth,
- low drift under Ψ-dynamics,
- reproducibility across seeds,

but necessarily fail τ-closure in the limit of refinement, i.e. $\Delta\tau \not\to 0$.

Corollary 1 (Derived τ-Structures)

Any τ-invariant constant not equal to $\sqrt{2}$, e, or π is a derived τ-structure whose stability depends on a composition of primary closure mechanisms.

[Uniqueness of the Basis] There exists no alternative set of constants that forms a smaller or distinct basis for τ-invariant closure classes within the UNNS substrate.

In particular, the existence of stable derived τ-structures (e.g. the golden ratio) does not constitute an extension of the basis, as such structures do not satisfy irreducible τ-closure.

5.10 Empirical Witness: φ-Class as a Derived τ-Structure (Operator XIV)

The paper asserts that non-basis constants may exhibit apparent stability while failing primariness because their closure mechanism is structurally composite. In Appendix A, the golden ratio is treated as a non-example precisely because its definition depends on $\sqrt{2}$-like closure behavior and iterative equilibration, hence failing irreducibility for primariness.[1]

[1] Appendix A: φ discussion.

We now attach an operational witness produced by *Chamber XIV (Operator XIV: Self-Contained with τ-Invariant Analysis)*. In this chamber, the measured φ-class output is tested for τ-closure by computing a closure residual $\Delta\tau$ and a closure signature.

[Closure Signature and Composite Closure] A *closure signature* is a finite tag that identifies which closure-mechanism families are implicated by a measured structure. A signature of length 1 denotes an irreducible closure mechanism (candidate for primariness); a signature of length > 1 denotes *composite closure* and therefore excludes primariness.

[Primariness Exclusion by Composite Signature] If a structure has a composite closure signature (length > 1), then it is not primary.

Primariness in the present paper requires irreducibility of the closure principle. A composite signature represents dependence on multiple closure-mechanism families and therefore violates irreducibility.

[Operator XIV witness for φ-class derivedness] In the Chamber XIV run reported here, the φ-class output is classified as a derived τ-structure with composite closure signature `OR`, and it fails τ-closure certification (nonzero closure residual $\Delta\tau$).

The exported Chamber XIV data show a closure signature `OR` and nonzero closure residual. For $\sigma = 0$, the run yields $\Delta\tau \approx 2.69 \times 10^{-2}$ with signature `OR` and `is_primary=false`. For $\sigma = 0.05$, the run yields $\Delta\tau \approx 4.96 \times 10^{-2}$ with signature `OR` and `is_primary=false`. By Lemma 5.10, composite signature excludes primariness.

[Operational support for Corollary 1] The φ-class is consistent with Corollary 1: it behaves as a derived τ-structure whose stability depends on composition of primary closure mechanisms rather than on an irreducible closure principle.

[Stability Without Primariness (Operational witness)] A structure may be reproducible and dynamically stable in the chamber while failing primariness whenever its closure signature is composite and its τ-closure residual does not vanish under refinement.

Corollary 2 (Uniqueness of the Basis)

There exists no alternative set of constants that forms a smaller or distinct basis for τ-invariant closure-classes within the UNNS substrate.

[Collapse Consistency] Operator XII preserves exactly the primary τ-invariant closures generated by $\sqrt{2}$, e, and π, while eliminating non-closed structures under refinement.

Derived τ-structures may persist for finite recursion depth and exhibit long effective half-lives, but are ultimately removed as $\Delta\tau$ remains bounded away from zero.

Corollary 3 (Collapse Consistency)

Operator XII does not alter the basis $\mathcal{B}_\tau$. It eliminates non-closed structures while preserving the closure-classes generated by $\sqrt{2}$, e, and π.

[Stability Without Primariness] Stability under recursion, reproducibility across seeds, and low dynamical drift are necessary but not sufficient conditions for τ-primariness.

A structure is primary if and only if it satisfies irreducible τ-closure under refinement.

Empirical validation chambers confirm that the golden ratio φ exemplifies a maximally stable derived τ-structure: reproducible, dynamically equilibrated, yet non-closed under refinement.

6 Derived τ-Structures

All remaining constants arise from compositions, interactions, or projections of the primary τ-invariants. Their stability is real and often empirically observable, but their structure is not irreducible.

They fail the primariness criterion by construction.

7 Conclusion

The UNNS substrate admits exactly three primary τ-invariants: $\sqrt{2}$, e, and π. They correspond to orthogonal closure, relaxation closure, and projection closure, respectively.

Operator XII does not generate these invariants. It reveals them by eliminating all non-closed structure. What persists is not numerical accident, but structural necessity

The following sections extend the framework without modifying the Basis Theorem or the primariness classification.

8 Composition Algebra of Derived Structures

The Basis Theorem establishes that $\sqrt{2}$, e, and π constitute *primary* τ-invariants: they arise as irreducible closure constraints under the $\Phi \to \Psi \to \tau$ progression and survive projection by Operator XII. All other constants encountered in mathematics and physics are therefore classified as *derived structures*.

To avoid destabilizing primariness, we do *not* introduce any operation that could generate new primary invariants. Instead, we define a *composition algebra* acting strictly on already established τ-invariants.

[Derived Structure] A *derived structure* is any quantity expressible as a finite or recursive composition of the primary τ-invariants $\{\sqrt{2}, e, \pi\}$ under admissible operations that preserve τ-closure but do not introduce new closure constraints.

Admissible operations include:

- multiplicative scaling,
- exponentiation with Φ-generated exponents,
- recursive limits whose attractor is already fixed by a primary invariant,
- projection-dependent ratios.

Constants such as the golden ratio φ or Apéry's constant $\zeta(3)$ are not excluded by this framework; however, their existence is explained as *composite stability*, not primariness. They inherit persistence from the primary invariants without originating new τ-closure.

9 Mechanics of Operator XII

Operator XII is defined as the terminal projection operator enforcing definiteness by eliminating structures that fail to maintain closure consistency under refinement.

While the Basis Theorem relies only on the survival or elimination outcome, further insight is gained by modeling the *dynamics* of collapse.

[Collapse Trajectory] A *collapse trajectory* is the evolution of a structure under repeated application of Operator XII as Φ-resolution increases.

[Structural Half-Life] The *structural half-life* of a structure is the number of collapse iterations required for its deviation from closure consistency to exceed a fixed threshold.

Primary τ-invariants have infinite structural half-life. Derived structures may exhibit long but finite half-lives, explaining why some non-primary constants appear robust across physical regimes.

This framework explains why certain derived quantities recur in nature without granting them foundational status. Their persistence is dynamical, not ontological.

10 Cross-Regime Mapping

The UNNS framework distinguishes three regimes of structure: Φ (discrete generation), Ψ (relational consistency), and τ (closure).

To clarify how structures traverse these regimes without creating new τ-invariants, we introduce a regime-mapping formalism.

[Regime Ascent Map] A *regime ascent map* is a rule that associates a Φ-generated sequence with a Ψ-level relation and determines whether this relation admits a τ-closure.

The Fibonacci sequence is a Φ-object. Its ratio convergence defines a Ψ-relation. However, the limit φ does not enforce a novel closure constraint; it remains expressible via recursive composition and therefore does not constitute a primary τ-invariant.

A discrete structure encounters the *threshold of transcendence* only when no finite Ψ-relation can satisfy closure without introducing a new irreducible constraint. This threshold is crossed only by $\sqrt{2}$, e, and π.

11 Extensions Without Basis Drift

This section extends the framework in three directions—(i) a composition algebra for derived τ-structures, (ii) collapse kinetics under Operator XII, and (iii) explicit cross-regime mapping $\Phi \to \Psi \to \tau$—while preserving the closed status of the primary τ-basis $\{\sqrt{2}, e, \pi\}$. No statement in this section introduces new primary τ-invariants or alters the Basis Theorem; instead, it formalizes the theory of *derived* structures and their empirical handling.

11.1 A. Composition Algebra of Derived τ-Structures

Definition (Derived τ-Structure)

A *derived τ-structure* is any stable quantity or relation whose persistence at the τ-regime depends on a composition of two or more primary closure mechanisms (orthogonal closure, relaxation closure, projection closure), possibly with additional regime parameters (e.g. projection choices).

Definition (Closure-Class Typing)

Let $\mathcal{B}_\tau = \{\sqrt{2}, e, \pi\}$ denote the primary τ-basis. A derived structure D is assigned a *closure signature* $\mathrm{sig}(D)$ taking values in the finite alphabet

$$\{\mathbf{O}, \mathbf{R}, \mathbf{P}\} \quad \text{(orthogonal, relaxation, projection)}$$

and their finite concatenations (e.g. **OR**, **OP**, **RP**, **ORP**), where the signature records *which closure mechanisms are required* for the stability of D.

Admissible Composition Operators (Regime-Safe)

We introduce three abstract composition operators acting on closure mechanisms:

- $\oplus$ (orthogonal composition): combines independent contributions into a single magnitude closure (orthogonal closure class).
- $\otimes$ (relaxation composition): combines sequential equilibration or rate-normalization steps (relaxation closure class).
- $\circ$ (projection composition): composes cyclic/rotational projection constraints (projection closure class).

These operators are *typed*: they operate on closure mechanisms, not on numerical constants as free parameters. A derived structure is admissible only if its closure signature can be constructed using $\oplus, \otimes, \circ$ over the basis mechanisms. Importantly, admissible composition *cannot create new primary closure types*; it can only produce composites.

Remark (Exponentiation and Limits)

Operations such as exponentiation, limits, or iterated recursion are permitted only as *derived constructors*. They may change the *degree* or *complexity* of a derived structure, but they do not define a new closure mechanism. Formally, any constructor must map closure signatures to closure signatures:

$$\mathrm{sig}(D) \mapsto \mathrm{sig}(F(D)) \in \{\mathbf{O}, \mathbf{R}, \mathbf{P}, \mathbf{OR}, \mathbf{OP}, \mathbf{RP}, \mathbf{ORP}\},$$

never outside this set.

Operational Goal

The algebra becomes predictive when a candidate constant is assigned a closure signature *before* numerical estimation: the framework predicts *which closure mechanisms must appear* for the candidate to remain stable at the τ-regime.

11.2 B. Operator XII Kinetics: Collapse as a Consistency Filter

Operator XII (Collapse / Sobra–Sobtra) is a destructive, terminal operator that eliminates unstable or residual recursive structures and returns recursion toward the Zero state. It does not generate closure; it removes what cannot remain consistent.

Definition (Closure Residual)

Let S denote a structure (sequence-state, constraint-state, or candidate closure configuration). Define a nonnegative scalar functional

$$\Delta_\tau(S) \geq 0$$

called the *closure residual*, measuring violation of the relevant τ-closure constraint (e.g. normalization defect, geometric closure defect, projection inconsistency).

Definition (Collapse Trajectory and Survival)

Define the collapse trajectory

$$S_{k+1} = \mathrm{XII}(S_k), \qquad k = 0, 1, 2, \ldots$$

A structure is said to *survive collapse* if the trajectory does not reach the Zero state and if $\Delta_\tau(S_k)$ remains bounded while the closure constraint remains identifiable.

Definition (Structural Half-Life Under Collapse)

Define the structural half-life of S under collapse as

$$t_{1/2}(S) = \min\{k \in \mathbb{N} \;:\; \Delta_\tau(S_k) \leq \tfrac{1}{2}\Delta_\tau(S_0)\},$$

with $t_{1/2}(S) = \infty$ if no such k exists. This quantity is an *internal collapse-iteration measure*, not physical time.

Interpretation

Primary τ-invariants are identified by persistence of closure through collapse. Derived structures can exhibit long half-life (large $t_{1/2}$) without being primary. This provides a principled explanation for why certain derived constants can appear "fundamental" in practice: they are *long-lived composites*, not basis elements.

11.3 C. Cross-Regime Mapping: $\Phi \to \Psi \to \tau$

To operationalize Φ–Ψ–τ progression, we define explicit regime lift maps.

Definition (Regime-Lift Maps)

Let X_Φ denote Φ-regime objects (finite approximants, enumerations, partial sequences). Let X_Ψ denote Ψ-regime relations (ratios, invariants, symmetries). Let X_τ denote τ-regime closure constraints (closure residuals).

Define two maps:

$$L_{\Phi\to\Psi} : X_\Phi \to X_\Psi, \qquad L_{\Psi\to\tau} : X_\Psi \to X_\tau.$$

The first extracts relational structure from discrete generators; the second evaluates closure defects and completion constraints.

Definition (Persistence Under Φ-Refinement)

Let $\{x_N\}_{N\geq 1}$ be a refinement sequence in X_Φ. A Ψ-relation R *persists under Φ-refinement* if

$$L_{\Phi\to\Psi}(x_N) \to R \quad \text{and} \quad \mathrm{drift}(L_{\Phi\to\Psi}(x_N)) \to 0$$

under refinement, for an explicitly chosen drift functional.

Definition (Threshold of Transcendence)

Define the threshold of transcendence N^* as the smallest refinement level such that both

$$\mathrm{drift}(L_{\Phi\to\Psi}(x_N)) \leq \varepsilon_\Psi \quad \text{and} \quad \Delta_\tau(L_{\Psi\to\tau}(L_{\Phi\to\Psi}(x_N))) \leq \varepsilon_\tau$$

hold for all $N \geq N^*$, where $\varepsilon_\Psi, \varepsilon_\tau$ are fixed tolerances. This provides an operational criterion for when a Φ-process "hits" the τ-regime.

11.4 Addressing Conceptual Challenges

1. Circularity Risk (Why Exactly Three?)

The Basis Theorem is a closure-class classification claim. To reduce circularity, the classification is not framed as "we observe three constants," but as "we identify three irreducible closure mechanisms". The mechanisms—orthogonal composition, equilibration, and projection—are defined structurally and are mutually irreducible by construction. Any proposed "fourth" type must be given as a new irreducible mechanism. If it decomposes into combinations of the three, it is not primary.

2. Operationalization (Making It Predictive)

Definitions in this section provide explicit functionals: Δ_τ (closure residual), drift (relation drift), and thresholds $(\varepsilon_\Psi, \varepsilon_\tau)$. These make the framework predictive in the sense that, given a candidate structure, one can pre-specify which residuals and drifts must stabilize for τ-persistence, and one can measure survival and half-life under collapse.

3. Empirical Connection (Why Nature Should Care)

The framework connects to empirical measurement through the following minimal bridge: measurement is modeled as truncation/projection that destroys unstable detail. Operator XII is the destructive filter representing this projection to definiteness. A closure constraint that persists across truncation scales, noise levels, and measurement regimes is empirically accessible as a stable structural residue. This is the sense in which τ-invariants are "physical": not as completed numbers, but as closure rules that the measurement process cannot eliminate.

4. Composition Algebra Incompleteness

The admissible operators $\oplus, \otimes, \circ$ are defined at the level of closure mechanisms. Constructors such as exponentiation, limits, and iterated recursion are allowed only when they preserve closure typing, i.e. they cannot introduce a new closure class beyond orthogonal/relaxation/projection. This prevents algebraic operations from masquerading as new primaries while preserving the generative nature of UNNS for derived structures.

Appendix A: Structural Non-Examples (Derived τ-Structures)

This appendix records representative constants that are often suspected to be fundamental, and explains why they do *not* qualify as primary τ-invariants under Definition 2.

A.1 The Golden Ratio φ

The golden ratio $\varphi = (1+\sqrt{5})/2$ arises from recursive ratio stabilization in linear recurrence relations. Its definition depends explicitly on $\sqrt{2}$-like closure behavior and iterative equilibration. Therefore φ is structurally composite and fails the irreducibility criterion for primariness.

A.2 Euler–Mascheroni Constant γ

The Euler–Mascheroni constant γ emerges as an asymptotic difference between harmonic sums and logarithmic growth. It is not associated with a closure principle that completes a structure, but rather with a residual offset in divergent behavior. Accordingly, γ does not define a τ-closure and is not a τ-invariant.

A.3 Feigenbaum Constants

The Feigenbaum constants characterize universality classes in period-doubling bifurcations. While they exhibit remarkable stability, they depend on parameterized dynamical systems and scaling limits. Their existence presupposes relaxation dynamics governed by e-like behavior, rendering them derived rather than primary.

A.4 Fine-Structure Constant α

The fine-structure constant α arises from the normalization of quantum couplings under specific projection regimes. Its numerical value depends on interaction structure, unit conventions, and renormalization context. As such, α is a projection-dependent derived τ-structure rather than an irreducible closure invariant.

Conclusion

Each non-example fails at least one criterion of Definition 2, typically by depending on combinations of orthogonal closure, relaxation, or projection. This confirms that $\sqrt{2}$, e, and π exhaust the class of primary τ-invariants.

Appendix B: Operator XII (Collapse / Sobra–Sobtra) — Codex Alignment

This appendix fixes the interpretation of Operator XII in strict accordance with the UNNS Operator Codex.

B.1 Status of Operator XII

Operator XII is a numbered UNNS operator. It is *not* a structural regime and must not be identified with Φ, Ψ, or τ. The Φ–Ψ–τ symbols denote organizational regimes of structure, whereas Operator XII acts on recursive states.

B.2 Functional Role

Operator XII is a destructive, terminal operator. Its function is to eliminate unstable, residual, or non-closed recursive structures and to return recursion toward the Zero state. Operator XII does not generate structure, does not define closure, and does not participate in the formation of τ-invariants.

B.3 Relation to τ-Invariants

τ-invariants are identified by their behavior under collapse. A structure qualifies as a τ-invariant only if it persists through the action of Operator XII. Operator XII therefore serves as a *consistency filter*, not as a source of invariance.

B.4 Canonical Statement

Operator XII reveals τ-invariants by destroying everything that is not closed.

This statement is definitive and exhausts the role of Operator XII within the present paper.